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Parametric Investigation and Classification of Quadratic Equation

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Abstract— This study investigated the papramters space of simple and familiar quadratic equation often encounter in science based disciplines and in particular engineering. The aim is strange images creation to excite and launch novices to quest for general understanding of fractal taking parallel clue from Julia and Mandelbrot sets creation using the iterative mechanisms of complex quadratic mapping. Preliminary computation of a Julia and Mandelbrot sets was performed on a square complex plane using coarse step size .Using normalization technique, the three parameters space of a general quadratic equation was reduced to two parameters space. Thereafter, several parameters combination drawn from within the range of two square parameters window specification was tested according to root values outcome using the popular 'Almighty formula'. The root values outcomes were classified into three distinct groups: complex roots, equal real roots and different real roots. Curve fitting,ratio and graphical analyses were performed on each group based on content and as applicable.The preliminary results of a Julia and Mandelbrot sets are acceptable to within the tolerance of coarse step size.The reduced two dimension parameters space was partioned by parabolic curve into two sub-blocks in favour of complex roots and different real roots outcomes.The sub-block in favour of different real roots dominate consistently over studied cases.The ratios of different real roots to complex roots gravitate respectively to 5.71 and 1.40 for longer and smaller windows with decreasing step size.The graphical analyses reveal strange fractal like images for cases investigated.The founding's of this study can be utilized to gently and safely launch both High School and College students to the world of fractal science,as the images generated are exciting and amazing..
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Index Terms— Fractal,Parametes Julia and Mandelbrot sets,Quadratic Equation,Root Values,Images Creation,Curve fitting,Ratio and Graphical Analyses

1 INTRODUCTION

THE importance of Quadratic equation as one of the basic equation in science and engineering cannot be overstressed. Quadratic equation has been employed as key equation for various modelling projects like satellite dishes. The path of a projectile (e.g cannon ball) is (almost) parabolic, and we use a quadratic equation to find out where the projectiles is going to hit (Bourne,2012).Parabolic antennas are another application.Real mathematics is about modelling situations that occur naturally, and using the model to understand what is happening, or even to predict what will happen in future.The quadratic equation is often used in modelling because it is a beautifully simple curve .In reality,quadratic equation has many functions in the scientific and mathematical world.The equation is used to find shapes,circles,ellipses, parabolas, and more.The quadratic equation can be used to model many different phenomena.For instance,it is possible to measure the height of a baseball as it was thrown straight up in the air and pulled down again. Numerous researchers have greatly applied the basic quadratic equation in solving science and engineering problems.

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According to Elliot (1972), rate equations with quadratic nonlinearities appear in many fields, such as chemical kinetics, population dynamics, transport theory, hydrodynamics, e.t.c. Such equations, which may arise from basic principles or which may be phenomenological, are generally solved by linearization and application of perturbation theory.Jared et al (2012) has developed an instability index theory for quadratic operator pencils acting on a a Hilbert space.In an extension of the known theory for linear pencils,explicit connections are made between the number of eigenvalues of a given quadratic operator pencil with positive real parts to spectral information about the individual operators comprising the coefficients of the spectral parameter in the pencil.More specifically,the authors considered the problem of the existence and stability of spatially periodic waves for the "good" Boussinesq equation. Analysis of the results of the instability index theory provides an explicit, and somewhat surprising, connection between the stability of a given periodic travelling wavesolution of the "good" Boussinesg equation and the stability of the same periodic profile, but with different wave speed, in the nonlinear dynamics of a related generalized Kortewegde Vries equation. A study has been made on the methods for dimension reduction and their application in nonlinear dynamics (Alois and Hans, 2001). .Linear and nonlinear Galerkin methods were compared in their efficiency in order to reduce infinite dimensional systems.In the implementation of the nonlinear Galerkin method using approximate inertia manifold theory, quadratic equation was applied in some sections.Jae-Hyeong and Won-Gil (2011) determined some stability stability results concerning the 2-dimensional vector variable quadratic functional equation in intuitionist fuzzy normed spaces(IFNS). The results obtained from the study showed that there is existence of a solution for any approximately 2-dimensional vector variable quadratic mapping.The authors concluded that the result obtained implies completeness of IFNS.Quadratic equation has become very useful for design and construction of engineering systems.Rodrigues et al (2011) paper proposes the design of a linear quadratic regulator (LQR) for a class of nonlinear systems modeled via norm-bounded linear differential inclusions(NLDIs).The system to be controlled was originally described by a nonlinear dynamical model in state-space form.By employing an approach based on the mean value theorem,the NLDI considers the nonlinear terms in the Taylor series expansion of the nonlinear equations as uncertainties in the model.The significance of Rodrigues et al paper is the construction of a procedure to design an LQR controller for the NLDI representation of the nonlinear system.Monica et al (2001) has also employed quadratic equation in his study of optimal algorithms for well-conditioned nonlinear systems of equations were solved by function optimization.The results of their study showed that the algorithm can be applied under certain assumptions to be optimized.This implies that an upper bound must exist for the norm of the Hessian whereas the norm of the gradient must be lower bounded.The results hold for systems of quadratic equations for which estimation for the requested bounds can be devised. In Yao (2012) paper,Quadratic equation was applied in tyre lifting mechanism of mathematical and mechanical knowledge,establishing geometric equations,establish the statics equation,calculation and analysis,solution binary quadratics and getting oil cylinder's thrust was extensively studied.El-Tawil and Al-Jihany (2008) paper rely on quadratic equation in the study of nonlinear oscillators under quadratic nonlinearity with stochastic inputs.Different methods were used to obtain first order approximations, namely ; the WHEP technique,the perturbation method,the Pickard approximations, the Adomian decompositions and the homotopy perturbation method (HPM).Triplett (2011) asserted that most mathematics students do not understand the potential of the discriminant.Usually discussed while learning the Quadratic formula to solve quadratic equations,students learn its use for the classification of type and number of solutions for each equation being solved.Few of the students are aware of many additional applications for the discriminant.Discriminant has been used extensively in quadratic formula to find solution of quadratic equations with parameters, solving linear differential equations with constant coefficients,investigating types of solutions for linear differential equations with variable coefficients,Applications in Number theory to Diophantine equations,classifying conic sections as well as solving Dynamic nonlinear systems problems.Despite the numerous applications of quadratic equation in science and engineering,many high school and college students is still a novice in its usage in the world of fractal science.This study employed quadratic equation in strange images creation to excite and launch novices to quest for general understanding of fractal science.

2 METHODOLOGY

2.1 Quadratic Equation

The function $f(x, A, B, C)$ consisting of one independent variable (x) and three parameters (A, B and C) as in equation (1) is called quadratic and often pose challenge to Mathematicians, scientists and Engineers in the course of their researches.The details discussion of Bairstow's iterative approach to roots findings of polynomial of order there and above by steven and Raymond (2006) shows that the concluding part of this method results in quotient that is quadratic.

$$f(x, A, B, C) = Ax^2 + Bx + C = 0 \tag{1}$$

Equation (1) can be re-written in a normalized form as in equation (2) in which $1 = A' \leftarrow \frac{A}{A}$, $B' \leftarrow \frac{B}{A}$ and $C' \leftarrow \frac{C}{A}$

$$f(x, A', B', C') = A'x^2 + B'x + C' = 0 \tag{2}$$

An 'Almighty formula' as in equation (3) can be used to prescribe appropriate independent variable (x) called roots that satisfy $f(x, A, B, C) = 0$ or $f(x, A', B', C') = 0$ or $f(x, B', C') = x^2 + B'x + C' = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{3}$$

Three distinct experiences spelled out by equations (4) to (8) are possible with the use of equation (3) due to $\sqrt{B^2 - 4AC}$ argument.

Two complex roots

This is due to $\sqrt{B^2 - 4AC} < 0$ and as such the two roots are respectively given by equations (3) and (4).

$$Root1 = x_1 = \frac{-B + \sqrt{|B^2 - 4AC|}}{2A} \tag{4}$$

$$Root2 = x_2 = \frac{-B - \sqrt{|B^2 - 4AC|}}{2A} \tag{5}$$

Two equal real roots

This is due to $\sqrt{B^2-4AC}=0$ and as such the two roots are same and respectively given by equation (5).

$$Root1 = Root2 = x_1 = x_2 = \frac{-B}{2A} \tag{6}$$

Two different real roots

This is due to $\sqrt{B^2-4AC}>0$ and as such the two roots are respectively given by equations (6) and (7).

$$Root1 = x_1 = \frac{-B+\sqrt{B^2-4AC}}{2A} \tag{7}$$

$$Root2 = x_2 = \frac{-B-\sqrt{B^2-4AC}}{2A} \tag{8}$$

The present study investigated the creation of geometrical patterns (strange or not) by use of collection of sets of (B',C') that leads to complex roots or equal real roots or different real roots following the good examples of Mandelbrot (1983), Barnsley *et al* (1989) and Devaney (1990). Similarly investigated were the geometrical patterns of the collections of corresponding associated roots values (*Root1, Root2*).

2.2 Parameters Range Studied

Though $-\infty \le B',C' \le \infty$ is opened to investigation the present study is limited in scope to square windows $-20 \le B',C' \le 20$ and $-2 \le B',C' \le 2$ and constant step size of 0.5 and 0.02 respectively. The case of square window $-2 \le B',C' \le 2$ is a zoomed form of the square window $-20 \le B',C' \le 20$ studied to check set details variation with increasing resolution.

2.3 Quadratic Equation in Engineering

Dynamics

The challenge in engineering to solve quadratic equations can arise in the quest for fundamental information such as eigenvalue about dynamic systems. Typical example is provided by second order differential governing equation (9) for a single degree of freedom excited dynamic system. The homogeneous or general solution of this equation deals with the case when the excitation is set to zero. The name general solution implies that the solution tell us something very fundamental about the system modelled by equation (9).-that is, how the system responds in the absence of external excitation. The general solution to all unforced linear single degree of freedom systems is of the form $y=e^{\lambda t}$. If this function is use in equation (9), the result is equation (10) or its reduced form equation (11). The result is a polynomial of order two or quadratic called the characteristic equation in λ . The roots of this polynomial are the values of - λ that satisfy equation (11) and the λ 's are referred to as the eigenvalues**.**

$$a_2\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_o y = F(t) \tag{9}$$

$$a_2\lambda^2 e^{\lambda t}+a_1\lambda e^{\lambda t}+a_o e^{\lambda t}=0 \tag{10}$$

$$a_2\lambda^2+a_1\lambda+a_o=0 \tag{11}$$

In term of qualitative assessment, equation (11) compare correspondingly well with equation (1) in which case $x\leftarrow\lambda; a_2\leftarrow A; a_1\leftarrow B$ and $a_o\leftarrow C$ .Thus the solutions seeking procedures are same.

Design of an Electrical Circuit

According to Steven and Raymond (2006) electrical engineers often use Kirchoff's laws to study the steady state behaviour of electric circuits including circuits that are transient in nature. Transient situation occur following circuit switch closure accompanied by adjustment period as a new steady state is reached. The adjustment period is closely related to the storage properties of the circuit elements and the phenomenon is well captured by the second order rate equation (12) involving charge (q). The circuit variables are voltage (V), current (i), resistor (R), inductor (L) and capacitor (C). A typical electrical engineering design problem mighty involves determination of proper resistor to dissipate energy at a specified rate, with known values for L and C. Equation (12) compares qualitatively with equation (9) for F (t) and mathematical analysis procedures are same.

$$L\frac{d^2q}{dt} + R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad (12)$$

Mandelbrot and Julia Sets

According to Mandelbrot (1983), Barnsley *et al* (1989) and Devaney (1990) these are fractals derivable from iteration of complex quadratic map given by equation (13) subject to the fulfilment of escape criteria given by equation (14). In equation (13) ,**C** is a complex constant of the form $C = a + ib$ in which the real and complex parts are respectively '**a**' and '**b**'. Each Z_{n+1} and Z_n are similarly defined. A Julia set is obtained for prescribed complex constant C and iterating all points on the complex plane ($-2 \le a, b \le 2$) one at a time according to (13). The collection of complex points that meets equation (14) after average of 40-iterations make Julia set. However Mandelbrot set is a sort of catalogue of the Julia sets associated with the complex quadratic mapping. It creates a map of the complex plane, which shows information about the Julia sets for each complex number C. That is a point Z on the complex plane is a member of Mandelbrot set if the Julia set for C=Z is connected (i.e. one continuous island in the complex plane). To create Mandelbrot's set start from $Z_0 = 0$ and iterate with all points on the complex plane one at a time as complex constant C. The collection of complex points ($-2 \le a, b \le 2$) that meets equation (14) after average of 40-iterations make Mandelbrot; set.

$$Z_{n+1} = Z_n^{\ 2} + C \qquad (13)$$

$$|Z_n| = \sqrt{a_n^{\ 2} + b_n^{\ 2}} \le 2 \,, \quad n \ge 40 \qquad (14)$$

3 RESULTS AND DISCUSSION

Though Mandelbrot (1983), Barnsley et al (1989) and Devaney (1990) gave abundant examples of fractal images derived from correct order of implementation of equations (13) and (14), figures 1 and 2 are samples obtained in the course of the present study to convince that bizarre images are reality with correct use of these equations.

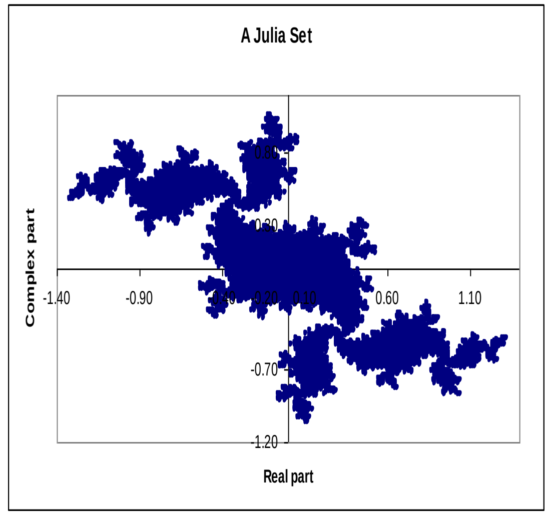

Fig. 1: A Julia set for C= - 0.2 + 0.75i in the Complex Plane -2 ≤ a, b ≤ 2 with constant step of 0.02

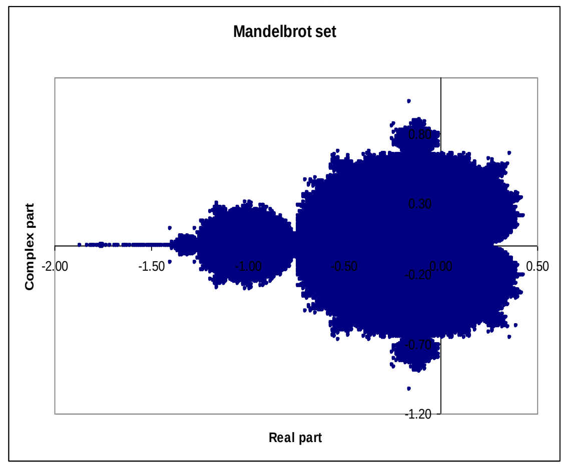

Fig. 2 : Mandelbrot set for $Z_0 = 0$ in the complex planes -2 ≤ a, b≤ 2 computed with constant step of 0.02

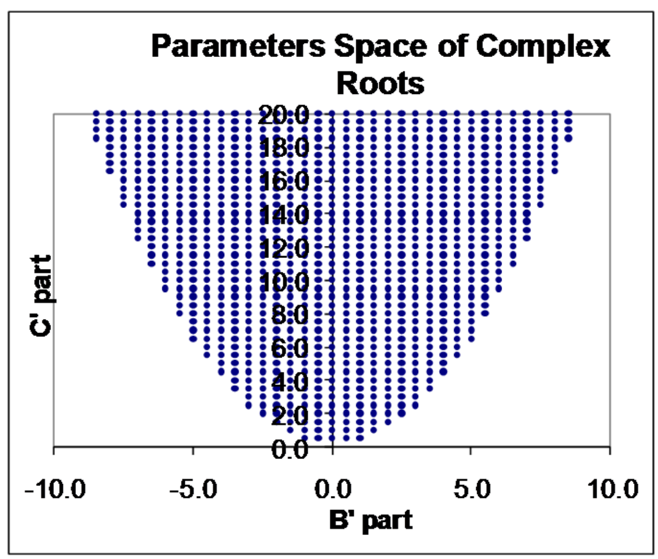

Fig.3: Parameter Space of Complex Roots in the $-20 \le B', C' \le 20$ Plane computed with constant step of 0.5

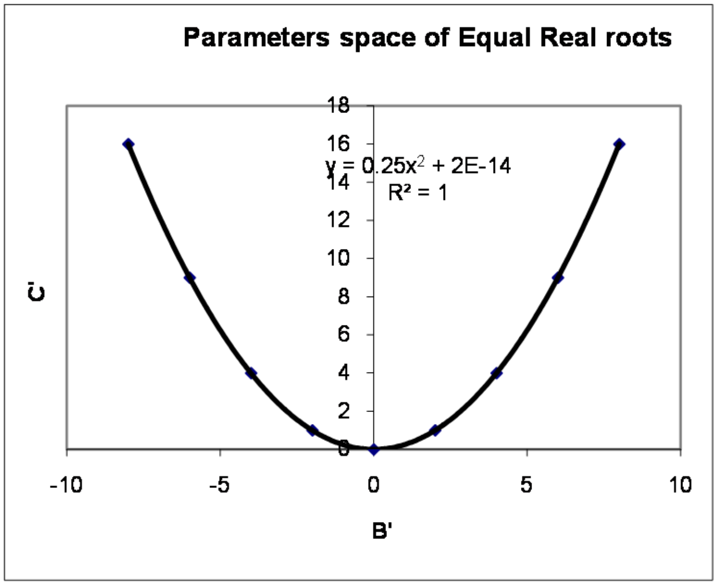

Fig .4 : Parameter Space of Equal Real Roots in the plane $-20 \le B', C' \le 20$ computed with constant step of 0.5

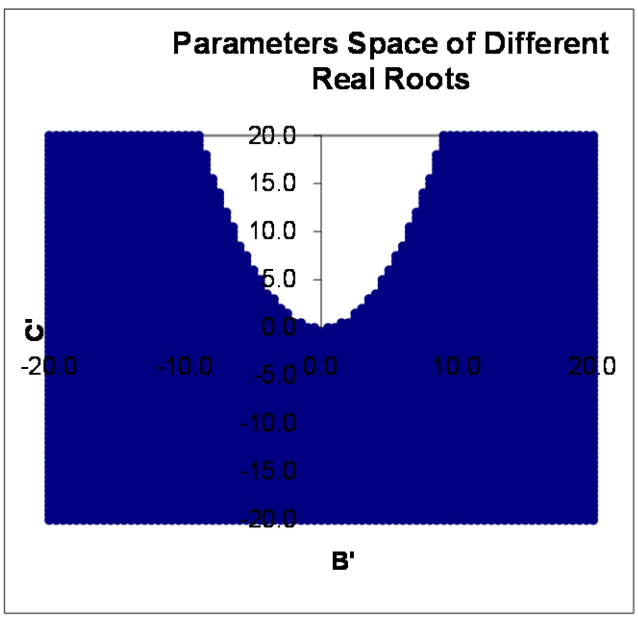

Fig.5: Parameters Space of Different Real Roots in the plane $-20 \le B', C' \le 20$ computed with constant step of 0.5.

Table 1: Variation of tested point's distribution in the plane $-20 \le B', C' \le 20$ with decreasing step size

Step size	Ratio of different real roots to complex roots	Total points tested	Number of points in favour of:		
			Complex roots	Equal real roots	Different real roots
0.50	5.83	6561	960	9	5592
0.25	5.76	25921	3834	13	22074
0.20	5.74	40401	5992	9	34400
0.10	5.72	160801	23912	9	136880
0.05	5.72	641601	95530	13	546058
0.02	5.71	4004001	596661	9	3407331
0.01	5.71	16008001	2385922	13	13622066

Table 2: Variation of tested point's distribution in the plane $-2 \le B', C' \le 2$ with decreasing step size

Step size	Ratio of different real roots to complex roots	Total points tested	Number of points in favour of:		
			Complex roots	Equal real roots	Different real roots
0.50	1.60	81	30	3	48
0.25	1.49	289	114	5	170
0.20	1.46	441	178	3	260
0.10	1.43	1681	690	3	988
0.05	1.42	6561	2714	5	3842
0.02	1.40	4040 1	16804	3	23594
0.01	1.40	1608 01	66945	5	93851

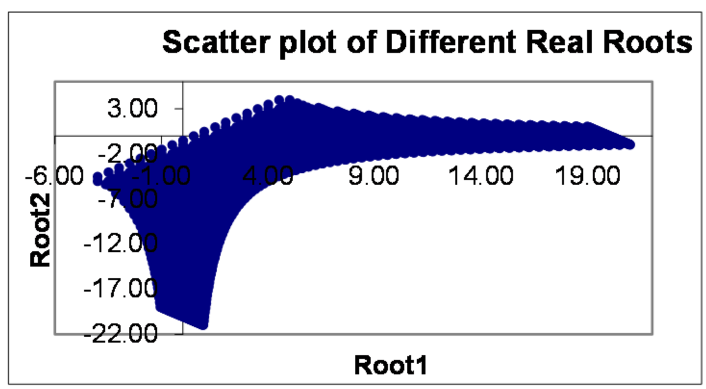

Fig.6: Scatter plot of Different Real Roots that correspond with parameters space $-20 \le B', C' \le 20$ in figure 5

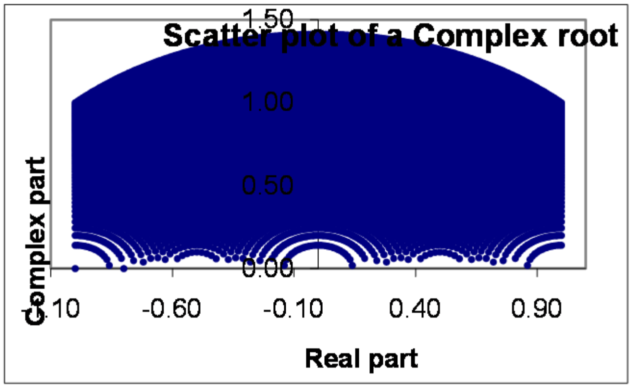

Fig.7: Scatter plot of a complex root that corresponds with parameters space $-2 \leq B^{'}, C^{'} \leq 2$ tested with step size of 0.02.

4 Conclusion

This study have shown that an arbitrary square block of quadratic equation parameters space can be partitioned by parabolic curve into two sub-blocks in favour of complex roots and different real roots outcomes. The sub-block in favour of different real roots dominate consistently over studied cases. Furthermore, this study has demonstrated the utility of simple and familiar quadratic equation as basis in creation of strange fractal like images. Thus with the founding's of this study, a gentle and safe launch of high School and College students to the world of fractal science is possible.

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